LOSS PROBABILITY APPROXIMATION OF A STATISTICAL MULTIPLEXER AND ITS APPLICATION TO CALL ADMISSION CONTROL IN HIGH-SPEED NETWORKS

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Abstract — This paper studies the cell loss probability approximation of a statistical multiplexer in high-speed networks. For this purpose, we consider discrete-time finite-buffer queueing models. We first propose a simple approximate formula for the cell loss probability in terms of the tail distribution of the queue length in the corresponding infinite-buffer queue. Since the tail distribution has a simple asymptotic expression in many situations, we revisit the asymptotic analysis of the tail distribution. Furthermore, we consider specific source traffic models which are particularly important in practice. The formula allows heterogeneous sources and each source is characterized only by a small number of parameters so that they meet engineering requirements. Combining these results, we propose a new call admission control (CAC) scheme which is based on the cell loss probability as a measure of quality of service.

I. INTRODUCTION

Bandwidth management in high-speed networks is considered as a key technology in providing services to such diverse traffic as video, voice and data. These traffic types have very different needs as far as quality of service (QoS) is concerned. In order to guarantee QoS to every transport connection established across the network, we need to have call admission control (CAC) which decides if a new call can be established with QoS required by the call while QoS of existing connections is not injured.

Typically, QoS is given in terms of the cell loss probability and delay. In this paper we consider the cell loss probability as QoS. In high-speed networks, the cell loss probability considered as QoS is very small, e.g., 10^{-7} or 10^{-10}. Thus, even if the exact analytical framework for the cell loss probability is available, we still have some difficulties in its computation. In particular, when many sources are accommodated in a multiplexer, the number of states to describe the dynamics of the multiplexer becomes prohibitively large. This fact makes the computation of the exact cell loss probability with enough accuracy very difficult. Furthermore, such an exact computation is time consuming. Thus, an efficiently computable yet accurate approximate formula of the cell loss probability should be developed.

In this paper, we study the cell loss probability approximation of a statistical multiplexer which is modeled by discrete-time finite-buffer queueing models with correlated arrivals. We assume that the arrival process is governed by a finite-state Markov chain. The mathematical models are described in section 2. In section 3, we propose a simple approximation formula of the cell loss probability, which is given in terms of the tail distribution of the queue length in the corresponding infinite-buffer queue. Therefore, once we obtain the tail distribution in the corresponding infinite-buffer queue, the cell loss probability can be estimated. In section 4, we revisit the asymptotic analysis of the tail distribution in the discrete-time infinite-buffer queue. Using the asymptotic formula as an approximation of the tail distribution, we consider the loss probability approximation when the arrival process comes from the superposition of many independent sources. In section 5, we discuss the application of the results to CAC in high-speed networks. We propose a new CAC scheme, which is based on the cell loss probability. In the past, several papers have discussed CAC, where the tail distribution itself is considered as a measure of QoS. As mentioned above, however, QoS is given in terms of the cell loss probability. Note that the approximate formulas of the cell loss probability show that the tail distribution can differ from the cell loss probability by more than one order of magnitude in some traffic conditions. Therefore, our CAC is expected to be an accurate control scheme which guarantees QoS given in terms of the cell loss probability.

II. QUEUEING MODELS

In this paper, a statistical multiplexer is modeled as a finite-buffer discrete-time single-server queue with correlated arrivals. Time is slotted, and the slot length is equal to a unit time. Cells arrive in batches. As for timings of arrivals, two queueing models have been explored: the early arrival model and the late arrival model (see p.5 of [12]). In the early arrival model, an arrival of a batch in the nth slot occurs immediately after the beginning of the nth slot. On the other hand, in the late arrival model, an arrival of a batch in the nth slot occurs immediately before the end of the nth slot. In what follows, we consider only the early arrival model (for the results of the late arrival model, see [7]).

Cell arrivals are governed by an underlying M-state Markov chain. This Markov chain changes its state on slot boundaries, and the state transition matrix for this underlying Markov chain is denoted by $U = \{U_{ij}\} (i, j = 1, \ldots, M)$, where we assume $U$ is irreducible. Let $P_r$ denote the state of the underlying Markov chain in the nth slot. Let $\pi = (\pi_1, \ldots, \pi_M)$ denote the stationary state vector of this Markov chain. Note that $\pi$ satisfies $\pi = \pi U$ and $\pi e = 1$, where $e$ is an M x 1 vector with all elements equal to one.

We now introduce some notations for the arrival process. Let $A_n$ denote the number of cells arriving in the nth slot. We assume that $A_{n+1}$ depends on both $P_n$ and $P_n$, see [2, 13, 14]. Let $A_{ij}(k)$ denote the conditional probability for the following events: $k$ cells arrive in the $(n+1)$st slot, and the underlying Markov chain is in state $j$ in the $(n+1)$st slot, given that the Markov chain was in state $i$ in the nth slot. Namely, $A_{ij}(k) = \Pr\{A_{n+1} = k, P_{n+1} = j | P_n = i\} (i, j = 1, \ldots, M)$. Let $A_k$ denote $M \times M$ matrices whose $(i,j)$th elements are given by $A_{ij}(k) (k \geq 0)$.

Our queueing system has finite buffer and accommodates at most N cells including the one in service. Thus, when $m (m \geq N - k + 1)$ cells arrive to find $k$ cells (including the one in service) in the system, only $N - k$ cells are accommodated in the system and the remaining $m - (N - k)$ cells are discarded.

The service time of a cell is assumed to be constant and is equal to a unit time. The service of a cell starts at the beginning of a slot and ends at the end of the slot. Cells depart from the system at slot boundaries.
Let \( \rho \) denote the traffic intensity which is given by \( \rho = \pi \sum_{k=1}^{\infty} kA_k e \).

### III. Cell Loss Probability Approximation

In this section, we show the formula for the exact cell loss probability and then propose a simple approximate formula for the cell loss probability in the early arrival model.

#### A. Exact Cell Loss Probability

Let \( y_k \) denote a \( 1 \times M \) vector whose \( j \)th element represents the joint stationary probability of \( k \) cells in the system and the underlyng Markov chain being in state \( j \). Let \( P_{\text{loss}} \) denote the cell loss probability.

Theorem 1. The cell loss probability in the early arrival model is given by:

\[
P_{\text{loss}} = \frac{\rho - (1 - y_0) e}{\rho}
\]

Proof. See [14].

#### B. Heuristic Approximation of the Cell Loss Probability

Let \( x_k \) denote a \( 1 \times M \) vector whose \( j \)th element represents the joint stationary probability of \( k \) cells in the system and the underlyng Markov chain being in state \( j \) in the corresponding discrete-time queue with infinite buffer. We suggest a conditional approximation of the number of cell in the system when the cell loss probability is very small:

\[
y_k \approx \frac{x_k}{\sum_{k=0}^{N} x_k e} \quad (0 \leq k \leq N).
\]

Remark 1. Similar conditional approximations have been studied by several researchers in the context of continuous-time queues. Readers are referred to [6, 8, 10, 15] and references therein.

We now propose the approximate cell loss probability using the approximation (1). We define the tail distribution \( T_k \) \( (k \geq 0) \) as

\[
T_k = \sum_{m=k+1}^{\infty} x_m e.
\]

Let \( \bar{P}_{\text{loss}} \) denote the approximate cell loss probability in the early arrival model. Noting the equalities \( \sum_{m=0}^{k} x_m e = 1 - T_k \) and \( x_0 e = 1 - \rho \), and using the approximation (1) in Theorem 1, we have

\[
\bar{P}_{\text{loss}} \approx \frac{(1 - \rho)T_N}{\rho(1 - T_N)}.
\]

Remark 2. When the number of cells arriving at the system is i.i.d., the above approximate formula becomes exact.

Remark 3. The approximate formula suggests that when the traffic intensity is very high or very low, the cell loss probability can differ from the tail distribution more than one order of magnitude.

#### C. Accuracy of the Approximation

We provide the results of our numerical experiments to show the accuracy of the proposed approximation. We assume that the cell arrival process is modulated by a two-state Markov chain with states 1 and 2, where the state transition probabilities \( U_{ij} \) are given by \( U_{11} = U_{22} = \alpha \) and \( U_{12} = U_{21} = 1 - \alpha \) \( (0 \leq \alpha < 1) \). If the Markov chain was in state 1 (resp. state 2) in the previous slot, the number of cells arriving in the current slot is geometrically distributed with the mean \( (1 + c)\rho \) (resp. \( (1 - c)\rho \)), where \( \rho \) denotes the overall traffic intensity, and \( c \) \((0 \leq c \leq 1)\) is a parameter. Note that, by keeping \( \rho \) and \( c \) constant, the correlation coefficient of the number of cells arriving in a slot depends only on the term \( 2c - 1 \). When \( \alpha = 0.5 \), the cell arrival process is i.i.d., and by varying \( \alpha \) from 0.5 to 1, we achieve varying degrees of non-negative correlations of arrivals. In the rest of this subsection, the tail distributions appeared in the approximate formula are computed by the matrix-analytic method [9, 13].

Table 1 shows the cell loss probability obtained by the approximate formula, the cell loss probability obtained by the exact analysis, and the relative error of the approximations to the exact results for various values of the buffer size \( N \), where the three parameters \( \rho = 0.8, \ c = 0.8, \ \alpha = 0.6 \). It is quite interesting to observe that the accuracy of the approximations is less sensitive to the buffer size \( N \). In [7], this fact has been investigated in more detail.

<table>
<thead>
<tr>
<th>( N )</th>
<th>exact</th>
<th>approximate</th>
<th>error(%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>3.065E-04</td>
<td>3.102E-04</td>
<td>1.191</td>
</tr>
<tr>
<td>70</td>
<td>2.320E-05</td>
<td>2.348E-05</td>
<td>1.189</td>
</tr>
<tr>
<td>90</td>
<td>1.756E-06</td>
<td>1.779E-06</td>
<td>1.189</td>
</tr>
<tr>
<td>110</td>
<td>1.332E-07</td>
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<td>1.189</td>
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<tr>
<td>130</td>
<td>1.010E-08</td>
<td>1.022E-08</td>
<td>1.189</td>
</tr>
<tr>
<td>150</td>
<td>7.652E-10</td>
<td>7.743E-10</td>
<td>1.191</td>
</tr>
<tr>
<td>170</td>
<td>5.798E-11</td>
<td>5.860E-11</td>
<td>1.209</td>
</tr>
</tbody>
</table>

Remark 4. The impacts of the correlation in the cell arrival process on the accuracy of the approximation have been also examined [7]. The examination has reported the following: When the correlation is not so strong, the approximation is surprisingly accurate. The error of the approximation becomes large according to the increase of the correlation. Even in those cases, we can use the approximation to estimate the order of magnitude of the cell loss probability.

### IV. Tail Distribution and Source Model

The approximate formula of the cell loss probability proposed in section 3 is given in terms of the tail distribution in the corresponding infinite-buffer queue. In general, it is, however, difficult to compute the tail distribution \( T_k \) because we must determine \( x_m \) \((m = k + 1, \cdots) \). Therefore we also consider approximating the tail distribution in an infinite-buffer discrete-time queue. Furthermore we consider applying the results to specific source traffic models.

#### A. Asymptotic Expression of the Tail Distribution

It has been shown that, in many queueing models, the tail distribution \( T_k \) has a rather simple asymptotic form [1, 4]. We can use the asymptotic formula as an approximation of the exact tail distribution in computing the approximate cell loss probability.

We define the matrix generating function (GF) for the arrival process: \( A(z) = \sum_{k=0}^{\infty} A_k z^k \). We assume \( \rho = \pi A(1/e) < 1 \) and the Markov chain \( A(1) \) is irreducible. Note that we can easily construct an arrival process in such a way that the queue length is bounded and has no tail. To ensure that the queue length has a simple asymptotic form, we need the following assumptions:

Assumption 1.

1. There exists at least one zero of \( \det z I - A(z) \) outside the unit disk.
2. Among those, there exists a real and positive zero \( z^* \), and the absolute values of \( z^* \) is strictly smaller than those of other zeros.
3. \( O < A(z) \ll +\infty, \ 1 \leq z \leq z^*, \ z \in R \).
It has been shown that the following holds under Assumption 1 [1, 4].

\[\text{Theorem 2.} \quad \text{Let denote } \delta(z) \text{ the Perron-Frobenius (PF) eigenvalue of the matrix } A(z) \text{, and } u(z) \text{ and } v(z) \text{ denote its associated left and right eigenvectors, which satisfy the normalizing conditions:}
\]
\[u(z)v(z) = 1, \quad (u(z)e = 1).
\]

For a sufficient large } k \text{, the tail distribution } T_k \text{ is approximately given by}
\[T_k \approx (1 - \rho)\frac{g}{\delta(z^*) - 1} \left(\frac{1}{z^*}\right)^k,
\]

where } z^* \text{ is a smallest positive root of } z - \delta(z) = 0 \text{ outside the unit disk and } g \text{ denotes a } 1 \times 1 \text{ vector, which is given by } g = 1/(1 - \rho)x_0 \text{ and can be obtained by solving a set of } M \text{ linear equations (see [9, 13]).}

\[\text{Proof.} \quad \text{See [7].}
\]

\[\text{B. Source Traffic Model}
\]

We consider an application of the results obtained so far to the case that the arrival process comes from the superposition of many sources. To this end, we first describe some important source traffic models, which were studied by Sohraby [11].

The most common source traffic model is a binary Markov source (BMS), where in any slot the BMS is in one of two different states. In the off-state, it does not generate a cell, and in the on-state, it generates only one cell. The transition probability from the on-state (resp. off-state) to the off-state (resp. on-state) is denoted by } 1 - \alpha \text{ (resp. } 1 - \beta) \text{, where } 0 \leq \alpha, \beta < 1. \text{ The matrix GF for the BMS is then given by}
\[
\begin{pmatrix}
\alpha z & 1 - \alpha \\
1 - \beta z & \beta
\end{pmatrix}.
\]

Note that the BMS is characterized by two parameters, } \rho \text{ and } B, \text{ which represents the traffic intensity and the mean burst length of a single BMS, respectively. It has been shown that [11]}
\[\alpha = 1 - 1/B, \quad \beta = 1 - \frac{\rho}{(1 - \rho)B}.
\]

For this BMS, the PF eigenvalue } \delta^*(z) \text{ and its associated left and right eigenvectors } \hat{u}^*(z) \text{ and } \hat{v}^*(z) \text{ are found to be}
\[\delta^*(z) = \frac{\alpha z + \beta}{2} + \sqrt{\left(\frac{\alpha z + \beta}{2}\right)^2 - \theta z},
\]
\[\hat{u}^*(z) = \frac{\delta^*(z) - \beta - 1 - \alpha}{1/\delta^*(z) - \theta},
\]
\[\hat{v}^*(z) = \frac{1}{2\delta^*(z) - \alpha - \beta} \left(\delta^*(z) - \theta, \delta^*(z) - \theta z\right)^T,
\]

where } \theta = \alpha + \beta - 1. \text{ Taking the first derivative of } \delta^*(z), \text{ we have}
\[\hat{\delta}'(z) = \frac{\alpha \delta^*(z) - (\alpha + \beta - 1)}{(2\delta^*(z) - \alpha - \beta)}.
\]

Note that the peak rate of the BMS is restricted to one. To overcome this difficulty, Sohraby has proposed the following source traffic model [11]. The source is constrained to generate a maximum of one cell every } R \text{ slots. Hereafter, we call this source model a GBMS (generalized BMS). The matrix GF for a GBMS with parameter } R \text{ is given by}
\[
\begin{pmatrix}
0 & 1 & 0 & 0 & \cdots & 0 & 0 \\
0 & 0 & 1 & 0 & \cdots & 0 & 0 \\
0 & 0 & 0 & 1 & \cdots & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
0 & 0 & 0 & 0 & \cdots & 1 - \alpha & 0 \\
0 & 0 & 0 & 0 & \cdots & 0 & 1 \\
\end{pmatrix}
\begin{pmatrix}
(R + 1) \\
R \\
\vdots \\
\alpha z \\
1 \\
1 \\
0 \\
0 \\
0
\end{pmatrix}
\begin{pmatrix}
1 \\
0 \\
0 \\
\vdots \\
1 \\
\beta \\
0 \\
0
\end{pmatrix}
\]

Note that the GBMS is characterized by three parameters, } \rho, B \text{ and } R, \text{ where}
\[\alpha = 1 - 1/B, \quad \beta = 1 - \frac{R\rho}{(1 - R\rho)B}.
\]

For the GBMS with parameter } R, \text{ the PF eigenvalue } \delta(z) \text{ and its associated left and right eigenvectors } \hat{u}(z) \text{ and } \hat{v}(z) \text{ are given in terms of } \delta^*(z), \hat{u}^*(z) \text{ and } \hat{v}^*(z):
\[\delta(z) = \delta^*(z)^{\frac{1}{R}},
\]
\[\hat{u}(z) = \delta^*(z)^{1/R} \hat{u}^*(z), \quad \hat{v}(z) = \delta^*(z) \hat{v}^*(z) \otimes \hat{\eta}(z),
\]

where
\[
\hat{\eta}(z) = \delta(z)^{R-1}(\delta(z) - 1)^T R \delta(z)^{R-1}(\delta(z) - 1)^T,
\]

and } \otimes \text{ denotes the Kronecker product [5]. Taking the first derivative of } \delta(z), \text{ we have}
\[\hat{\delta}'(z) = \frac{\alpha \delta^*(z) - (\alpha + \beta - 1)}{R \delta(z)^{R-1}(\delta(z) - 1)^T}.
\]

\[C. \text{ Superposition of Independent BMS's}
\]

We now consider the superposition of } K \text{ independent BM-S's. We assume that the } i \text{th source (} i = 1, \ldots, K) \text{ is characterized by } \alpha(i) \text{ and } \beta(i). \text{ In what follows, we assume } \rho = \sum_{i=1}^{K} \rho(i) < 1, \text{ where } \rho(i) \text{ denotes the traffic intensity of the } i \text{th source. We denote by } \delta(i)(z) \text{ the PF eigenvalue for the } i \text{th source, and by } \hat{u}^{(i)}(z) \text{ and } \hat{v}^{(i)}(z), \text{ its associated left and right eigenvector with normalizing conditions } \hat{u}^{(i)}(z) \hat{v}^{(i)}(z) = \hat{u}^{(i)}(z)e = 1. \text{ The PF eigenvalue } \delta(z) \text{ for the superposed arrival process is then given by}
\[\delta(z) = \prod_{i=1}^{K} \delta(i)(z), \tag{3}
\]

and its associated left and right eigenvectors } u(z) \text{ and } v(z) \text{ are given by}
\[u(z) = \hat{u}^{(1)}(z) \otimes \hat{u}^{(2)}(z) \otimes \cdots \otimes \hat{u}^{(K)}(z), \tag{4}
\]
\[v(z) = \hat{v}^{(1)}(z) \otimes \hat{v}^{(2)}(z) \otimes \cdots \otimes \hat{v}^{(K)}(z). \tag{5}
\]

Using Theorem 2, we have the following theorem.
Theorem 3. In the queue with the superposition of $K$ independent GBMS’s, the tail distribution $T_k$ is approximately given by

$$T_k = (1 - \rho) \frac{c(z^*)}{\delta(z^*) - 1} \left( \frac{1}{z^*} \right)^k,$$

where $z^*$ is a smallest positive real root of $z - \delta(z) = 0$ outside the unit disk, $c(z)$ is given by

$$c(z) = \prod_{i=1}^K \frac{\tilde{\delta}(i)(z) - (\alpha(i) + \beta(i) - 1) z}{2 \tilde{\delta}(i)(z) - \alpha(i) z - \beta(i)},$$

and $\delta'(z^*)$ is computed by

$$\delta'(z^*) = z^* \sum_{i=1}^K \tilde{\delta}(i) \frac{\delta'(z^*)}{\tilde{\delta}(i)(z^*)}.$$

Proof. See [7].

D. Superposition of Independent GBMS’s

We consider the superposition of $K$ independent GBMS’s. We assume that the $i$th source is characterized by $\alpha(i), \beta(i)$ and $R(i)$. In what follows, we assume $\rho = \sum_{i=1}^K \rho(i) < 1$ and $\sum_{i=1}^K 1/R(i) > 1$ so that the queue length tail exists [11].

We denote by $\tilde{\delta}(i)(z)$ the PF eigenvalue for the $i$th source, and by $\tilde{\delta}(i)(z)$ and $\tilde{\delta}(i)(z) = (\tilde{\delta}(i)(z), \ldots, \tilde{\delta}(i)(z))$, its associated left and right eigenvector with normalizing conditions $\tilde{u}(i)(z)\tilde{v}(i)(z) = 1$. Then the PF eigenvalue $\delta(z)$ and its associated left and right eigenvector $u(z)$ and $v(z)$ for the superposed arrival process are given in (3), (4) and (5), respectively. Contrary to the case of the BMS, we need to compute $g$ in order to obtain the exact asymptotic tail distribution. Using Theorem 2, however, we have the upper and lower bounds for the tail distribution.

Theorem 4. In the queue with the superposition of $K$ independent GBMS’s, the tail distribution $T_k$ is bounded for a sufficient large $k$ as

$$(1 - \rho) \frac{c_{\min}(z^*)}{\delta'(z^*) - 1} \left( \frac{1}{z^*} \right)^k \leq T_k \leq (1 - \rho) \frac{c_{\max}(z^*)}{\delta'(z^*) - 1} \left( \frac{1}{z^*} \right)^k,$$

where $\delta'(z^*)$ is computed by (7) and

$$c_{\min}(z) = \prod_{i=1}^K \min_{j \neq 1} [c(i)(z)]_j, \quad c_{\max}(z) = \frac{1}{1 - \rho} \prod_{i=1}^K \tilde{\delta}(i)(z)$$

with $\theta(i) = \alpha(i) + \beta(i) - 1$ and for $z > 1$

$$\min_{j \neq 1} \frac{\tilde{\delta}(i)(z)}{R(i)^{\gamma(i)}(z)} = \begin{cases} \frac{\tilde{\delta}(i)(z)}{R(i)^{\gamma(i)}(z)} - 1 (\text{if } \theta(i) > 0), \\ \min(\tilde{\delta}(i)(z), \tilde{\delta}(i)(z)) (\text{if } \theta(i) = 0), \\ \min(\tilde{\delta}(i)(z), \tilde{\delta}(i)(z)) (\text{if } \theta(i) < 0), \end{cases}$$

and

$$\tilde{\delta}(i)(z) = \begin{cases} \frac{1}{1 - \theta(i)} (0, 1 - \alpha(i)) (\text{if } R(i) = 1), \\ \frac{1}{R(i)^{\gamma(i)}(1 - \theta(i))} (0, 1 - \beta(i), 1 - \beta(i), 1 - \beta(i), 1 - \alpha(i), 1 - \alpha(i)) (\text{if } R(i) > 1). \end{cases}$$

Proof. See [7].

V. APPLICATION TO CAC

We now consider CAC in high speed networks. Our CAC scheme is based on the cell loss probability, rather than the tail distribution. We assume here that when a new call requires a connection, it declares the peak rate, the traffic intensity and the mean burst length. The new call is accepted if the resulting loss probability is less than a small number $\epsilon$ for a given buffer size $N$. Note that in high speed networks, we need to decide in real time if a new call can be established. To do so, we propose a closed form approximate formula of the cell loss probability for a sufficient large $N$, where we assume many sources are multiplexed and each source is characterized as a GBMS. Recall that the approximate loss probabilities (2) are given only in terms of the tail distribution. Thus the problem is reduced to the approximation of the tail distribution.

A. Approximations of the Tail Distribution

To obtain the tail distribution numerically, we need to compute a positive real root $z^*$, which can be computed by some common numerical algorithm. Sohraby has proposed the following approximation to $z^*$ [11]:

$$z^* \approx 1 + \frac{1 - \rho}{\sum_{i=1}^K \rho(i)(1 - R(i)^{\gamma(i)}(1 - \theta(i))^2)}$$

where $\rho(i), E(i)$ and $R(i)$ denote the traffic intensity, the mean burst length and the inverse of the peak rate of the $i$th source, respectively. In what follows, we use (8) as an approximation of $z^*$.

When all sources are BMS’s, an immediate approximation to $T_k$ is given by (6) and (8). When some of sources are GBMS’s, however, we need to evaluate the vector $g$, or the coefficient of $(1/z)^k$. In the past, several researchers have discussed tail distribution and CAC using

$$T_k \approx (1/z)^k,$$

which is the basis of the effective bandwidths technique (see [16] and references therein). On the other hand, Sohraby has proposed an approximation $T_k$ to the tail probability $T_k$ [11]:

$$T_k \approx \rho(1/z^*)^k.$$

It is known, however, that the coefficient of $(1/z)^k$ can be much smaller than the above, especially when the number of multiplexed sources is large [3]. Thus, further investigations should be made.

Since we have the upper and lower bounds of the coefficient of $(1/z)^k$ in Theorem 4, we may take a geometrical mean $T_k$ of the upper and lower bounds as an approximation to the coefficient:

$$T_k \approx (1 - \rho) c_{\min}(z^*)^{1/2} c_{\max}(z^*)^{1/2} (1/z)^k,$$

where $z^*$ is given by (8).

From our numerical experiments, however, we found that the lower (resp. upper) bound decreases rapidly (resp. slowly) as the peak rates of sources become small. This observation suggests that the lower bound can differ from the upper bound more than a few orders of magnitude in some situations. In such a case, the contribution of the lower bound to the geometrical mean may be too strong. Therefore, we propose another approximation $T_k$ of the tail probability:

$$T_k \approx (T_k^{\geo})^{1/2} (T_k^{\geo})^{1/2}.$$

The accuracy of these approximations is examined in the next subsection.
B. Numerical Examples

Now we show some numerical examples. In Fig 1, we assume that all sources are homogeneous, the buffer size $N = 200$, the total traffic intensity $\rho = 0.7$ and the mean burst length $B = 50$. The number of sources $K$ and the peak rate $R^{-1}$ are set to $4n$ and $1/(2n)$, respectively, where $n \geq 1$ is an integer parameter. Note that in these settings, both the exact value of $z^*$ and its approximation (8) are independent of a parameter $n$. Fig 1 shows the cell loss probability approximation as a function of a parameter $n$. We observe the followings from Fig 1. As the increase of a parameter $n$ (i.e., the increase of the number $K$ of sources while keeping $z^*$ fixed), the cell loss probability approximation with the lower bound decreases rapidly. On the other hand, the cell loss probability approximation with the upper bound gives the upper bound of the cell loss probability in a wide range, but the difference between the approximate and simulation results is about one order of magnitude. The cell loss probability approximation with the geometrical mean of the upper and lower bounds does not work well when a parameter $n$ is large. This is due to the fact that the lower bound of the tail distribution decreases rapidly as the increase of a parameter $n$. Note that when the differences between the approximations with the upper and lower bounds are less than two orders of magnitude, the approximation with the geometrical mean gives an accurate approximation. The cell loss probability approximation with (9) has an almost fixed relative error to the simulation results in all range of a parameter $n$.

Figure 2: Approximation and Simulation Results

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REFERENCES


